Discounted Emerging Cost Techniques and Profit Testing

SDM Profit Testing

If you only have one decrement, such as death then the way a cashflow emerges in a pure assurance policy will be as follows:

As you can see there is only one decrement here: DEATH

What if we add a survival benefit:

[The sum B is paid each year on survival]

? *Is this a multiple decrement model*

? *What would make this a multiple decrement model*

The Second Decrement

If we add withdrawal to our model and we say that the withdrawal benefit each year is W_t, can we then amend our formula as follows:

$$CF_t = P_t - e_t + i(P_t - e_t) - SA.\, q_{x+t-1} - B.\, p_{x+t-1} - W_t.\, q^w_{x+t-1}$$

? *What is the correct way to write the cashflow formula given we now have two decrements:*

The above formula assumes the sum assured is fixed, that there is a fixed survival (renewal) benefit each year and that the withdrawal benefit varies each year, but this all depends on how each policy is defined.

What is important in the above is the use of MDM probabilities

Exam Style Question

A five year endowment assurance policy, issued to 60 year old men, has premiums of £1000 per year and a sum assured of £5000 payable at the end of the year of death, or on survival to the five year term

The expenses are £100 at the start and then £20 on the payment of the next premium which then increases by 5% per year. You may assume interest rates are 3%.

If the member withdraws from the scheme he receives 50% of all the premiums back without interest

The population experience mortality given by the following table:

x	l_x
60	100000
61	99000
62	97800
63	96300
64	94600
65	93000

The independent decrement rate for withdrawals is given in the following table

x	q_x^w
60	0.3
61	0.2
62	0.2
63	0.1
64	0.01
65	N/A

Produce the full proft test for this policy including the cashflows, expected cashflows and profit vector (at a risk discount rate of 7%)

STEP 1: we need a multiple decrement table

x	l_x	q_x^w	μ_x^d	μ_x^w	$(aq)_x$	$(al)_x$	$(ad)_x$	$(ad)_x^d$	$(ad)_x^w$
60	100,000	0.3	0.01005	0.35667	0.30700	100,000.0	30,700.0	841.4	29,858.6
61	99,000	0.2	0.01220	0.22314	0.20970	69,300.0	14,532.0	753.0	13,779.0
62	97,800	0.2	0.01546	0.22314	0.21227	54,768.0	11,625.6	753.1	10,872.5
63	96,300	0.1	0.01781	0.10536	0.11589	43,142.4	4,999.7	723.0	4,276.7
64	94,600	0.01	0.01706	0.01005	0.02674	38,142.7	1,020.1	641.9	378.2
65	93,000	N/A				37,122.6			

What are the formula we have used:

We can also now easily get our multiple decrement probabilities fom the formula $(aq)_x^d = \frac{(ad)_x^d}{(al)_x}$

We are now ready to produce our profit testing spreadsheet:

x	P_t	e_t	$i(P_t - e_t)$	$D_t.(aq)_x^d$	$W_t.(aq)_x^w$	$S_t.(ap)_x$	CF_t	${}_t(ap)_x$	$E(CF_t)$
60	1000	100.00	27.00	42.07	149.29	0	735.64	1.0000	735.64
61	1000	20.00	29.40	54.33	198.83	0	756.24	0.6930	524.07
62	1000	21.00	29.37	68.75	297.78	0	641.84	0.5477	351.52
63	1000	22.05	29.34	83.79	198.26	0	725.24	0.4314	312.89
64	1000	23.15	29.31	84.14	24.79	4,866.28	-3,969.06	0.3814	-1,513.91

The above may seem counter intuitive at first

This is because actuaries are used to wrapping everything up in present values. Here we are looking at cashflows.

You need to read across the table and think about actual money to really understand this.

Imagine your bank balance just going up and down:

At time $t = 0$ premium P_t comes in and expenses e_t go out

Over year 1 interest at rate i on the bank balance of $P_t - e_t$ accrues to the company

At then end of the year benefits have to be paid of, for example, whatever the death benefit is D_t times the chance that a death has actually occured: $(aq)_x^d$

CF_t is then the sum of all the cashflows for the year

If we multiply CF_t by the chance that a policyholder made it to the start of the year ${}_t(ap)_x$ we get $E(CF_t)$ which is the expected cashflow for the year.

spreadsheet is here

From Cashflows to Profit

? *Could you take the above spreadsheet and calculate if this is expected to be a profitable policy*

? *Would it be fair to say the above policy made profits in years 1 to 4 and then a loss in year 5*

? *To get to the formula for the 'profit vector' we need to consider:*

- At the start of year t there will already be some reserves built up ${}_{t-1}V$
- Over the course of the year the reserves will receive interest $i.{}_{t-1}V$
- At the end of the year we will need to set aside a reserve for the next year. ${}_tV$ if we survive the year

? *So what is our formula for profit*

$PRO_t = CF_t$

You may often see this written as:

$PRO_t = CF_t + i.{}_{t-1}V - IR_t$ where $IR_t = (ap)_{x+t-1}.{}_tV - {}_{t-1}V$

Profit Signature

You may notice we appear to have lost the chance of making it to the start of the year again in the definition of PRO_t

Profit signature σ_t solves this

$\sigma_t =_{t-1} (ap)_x \times PRO_t$

We CAN now say the present value of future profits is:
$PVFP = \sum_{t=1}^{n} \frac{\sigma_t}{(1+r)^t}$

Isn't this the same as: Present value (Expected cashflows)$= \sum_{t=1}^{n} \frac{E(CF_t)}{(1+r)^t}$

Note: As reserves are difficult to calculate for MDM an exam question on profit signature would typically give you the reserve each year and so should be relatively straightforward

Interest Rates

You may have noticed that we seem to have several different interest rates now:

r is the risk discount rate at which we discount the profits

i is the return we expect to get on our reserves each year and also on the premiums that are paid at the start of the year

There will also be i_π which is the rate used in the premium basis and possibly also i_v the rate of interest implicit in the reserving basis

Discussion

How would you decide how to set these different rates of interest in practice

Profit Criteria

Commonly used criteria are:

The present value of expected future profits

$$PVFP = \sum_{t=1}^{n} \frac{\sigma_t}{(1+r)^t}$$

The Profit Margin

This is the present value of the expected future profits divided by the present value of the expected future premiums, both calculated at the same interest rate, i_m say

$$PM(i_m) = \frac{\sum_{t=1}^{n} \sigma_t . (1+i_m)^{-t}}{\sum_{t=1}^{n} P_{t \cdot t-1} (ap)_x . (1+i_m)^{-(t-1)}}$$

The Discounted Payback Period

This is the lowest value of k for which $PVFP = \sum_{t=1}^{k} \frac{\sigma_t}{(1+r)^t} \geq 0$

Internal rate of return

This is the rate of interest at which the profit signature has a present value of zero.

In many practical situations, a reduction in the size of the reserves leads to an increase in the internal rate of return.

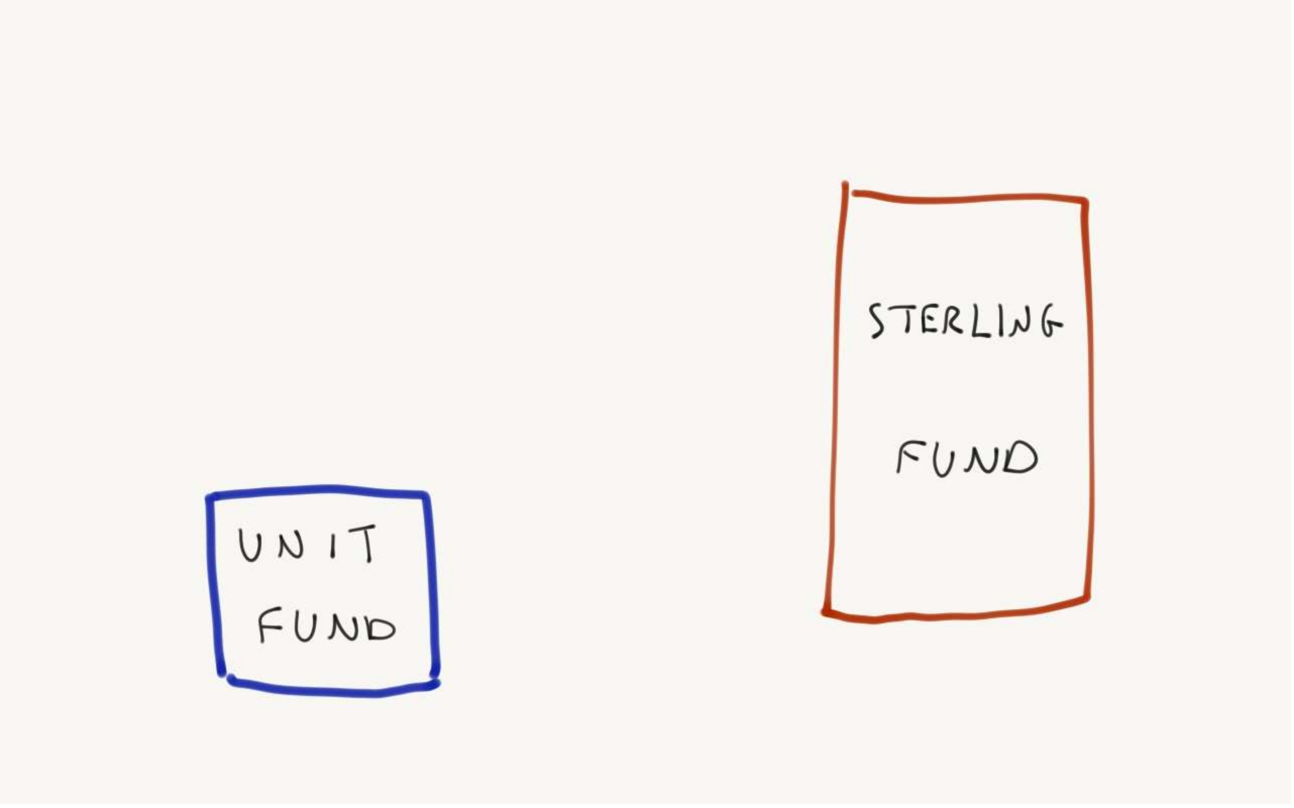
Profit Testing: Unit Linked Policies

Unit linked polices are fundementally different from normal life assurance polices as the policyholder receives whatever return the unit funds produce.

This has a number of consequences

- Funds can be invested in riskier assets to produce a higher return as the company does not have specific bonuses to meet
- The company still has to pay expenses, make a profit and pay specific assurance benefits
- The overall structure can become complicated as there needs to be separate funds: the unit fund itself and the sterling fund (which is the company's money)

The diagram below shows how this sort of structure will typically look:



0 1 2 3 4 5 6 7 8

In unit linked policies there are two funds. One is effectively the policyholder's fund and the other is the company's

All the money comes from the premiums.

Some of the premiums then go straight into the STERLING FUND and are effectively charges

Of the rest, there is then a BID/OFFER spread (which is another charge)

Each year there is then a further charge on the fund which is a proportion of the value of all the units.

The sterling fund then also increases with interest

The company's expenses then have to be paid from the sterling fund

Benefits (death, surrender and maturity) are then paid from the unit fund, with any shortfall made up from the sterling fund.

The profit is then whatever is left in teh sterling fund at the end.

Example

The sort of information you may need to analyse unit funds is as follows:

Policy definition

Premiums: £1000 per year

Term: 5 years

50% of units are allocated in the first year and 99% in the following years

There is a bid offer spread of 5% on the purchase of the units and an anual management charge of 1% is levied on the value of the unit fund at the end of the year

Assumptions

Initial expenses will be £150 and then £30 at the start of each subsequent year then increasing by 5% per year

The return on the unit fund will be 7% per year and the return on the sterling fund will be 2% per year

The policy is issued to a male life aged 60 who experiences mortality of $q_x = 0.01$ for the next 5 years

There is a sum assured of £5000 payable at the end of the year of death

What happens

This computer program allows you to see where each cashflow actually goes

This spreadsheet shows you the sort of calculations that you may need to perform in an actual exam question

Breaking it Down

The way to think of this situation is to look at each premium and then decide which pot it goes into

A premium of P is paid

Pa is allocated to the unit fund and goes straight into the sterling fund. These are not actually unallocated units - it is just money that goes into the insurance company's reserves.

The Pa does not go into the unit fund as there is still the bid offer spread of λ to consider.

So the amount that goes into the unit fund is and the amount that goes into the sterling fund is a further

For both the unit and the sterling funds this is clearly added to what is already there

The expenses then have to come out of the STERLING FUND.

> The insurance company does not pay rents and salaries out of the unit fund. It takes fees and charges out of the unit fund and then pays its expenses out of its 'own' sterling fund

A year passes and

- the sterling fund will typically receive a low and secure return as it will be invested in short term bond like assets
- the unit fund will receive a more volatile return as it is likely to be invested in more equity like assets

The annual management charge (AMC) is then paid from to (as this is a fee for management of the assets)

If the policyholder has died over the year then the sum assured is paid to the policyholder by the insurance company. The insurance company will use all the funds in the unit fund to pay this and will then use its own sterling fund to top the amount up to the full sum assured value

So the cost of providing the death benefit is where U_t is the unit fund value at time t

Profit

The profit of the life assurance company is now just how much money it has left at the end in the sterling fund

The unit fund all goes to the policyholder

When considering standard L.A. polices we do not just consider the money left at the end so we should extend the argument in an analagous way for unit linked funds

Options for defining profit

- Just use the sterling fund at the end of the policy term
- Use the discounted expected cashflows into the sterling fund for each year
- Adjust the cashflows each year by the change in reserves required to fund for any expected negative future cashflows

These methods work the same as standard policies - so we don't need to break down all the algebra here.

The distinctions for unit linked polcies are less important though because

A common method is to set up a reserve in the first year so that the profit signature after year 1 is never negative. This is called ***zeroisation***

Zeroisation

The point of ***Zeroisation*** is to remove negative values of the profit signature ($E(CF_t)$) after year 1, by setting aside a reserve in year 1.

Look at the table below

year	age	fees	expenses	reserve	interest	AMC	death benefit	CF	in force prob	profit signature
1	60	200	100	-	4.00	10	130	-16.00	1.000	-16.000
2	61	80	10	-	2.80	20	112	-19.20	0.987	-18.950
3	62	60	10	-	2.00	30	90	-8.00	0.973	-7.785
4	63	60	10	-	2.00	40	64	28.00	0.959	26.840
5	64	60	10	-	2.00	50	34	68.00	0.943	64.141

In which we assume $i_s = 4\%$ and the sum assured is 10000. By i_s we mean the return on the sterling fund as this is the return we get on the reserves we are effectively 'setting aside'.

Mortality is as given below:

Age	q_x
60	0.013
61	0.014
62	0.015
63	0.016
64	0.017

How much reserves do we need to hold at the start of year 3, so that the profit signature for year 3 is zero

There are two things to consider here:

- We will receive another year's interest before we 'make the loss' next year AND
- only ${}_2p_{60}$ policies are in force at the start of the year

Note: While it is possible to do the calculation directly in terms of profit signature - it is easier to understand in terms of the ***cashflow vector*** as then you are only having to put probabilities in rather than having to back them out as well.

So the reserve we need at the start of year 3 is

? *Does this mean we need to reduce the cashflow vector by this amount for year 2*

How much do we need to reduce the year 2 cashflow

So the steps are:

- Take the negative cashflow from year 3
- Back out the interest earned during year 3
- Reduce by the ratio of in-force probability at start year 3 to in-force probability start year 2

So the adjusted cashflow for year 2 becomes:

We now have the following adjusted cashflow vector:

t	CF_t
1	-16.00
2	-26.78
3	-
4	28.00
5	68.00

We repeat the process in exactly the same way to remove the -26.78 figure

So adjusted cashflow vector for year 1 becomes:

So the final zeroised cashflow vector becomes

t	CF_t
1	
2	
3	
4	
5	

From which we can then also work out the profit signature in the obvious way

t	CF_t	σ
1		
2		
3		
4		
5		